

GROUND STATE MASS SPECTRUM FOR SCALAR DIQUARKS WITH BETHE-SALPETER EQUATION

Z. G. Wang¹, S. L. Wan² and W. M. Yang²

¹ Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

² Department of Modern Physics, University of Science and Technology of China, Hefei 230026, P. R. China

Abstract

In this article, we study the structures of the pseudoscalar mesons π , K and the scalar diquarks U^a , D^a , S^a in the framework of the coupled rainbow Schwinger-Dyson equation and ladder Bethe-Salpeter equation with the confining effective potential. The u , d , s quarks have small current masses, and the renormalization is very large, the mass poles in the timelike region are absent which implements confinement naturally. The Bethe-Salpeter wavefunctions of the pseudoscalar mesons π , K and the scalar diquarks U^a , D^a , S^a have the same type (Gaussian type) momentum dependence, center around zero momentum and extend to the energy scale about $q^2 = 1\text{GeV}^2$ which happen to be the energy scale for the chiral symmetry breaking, the strong interactions in the infrared region result in bound (or quasi-bound) states. The numerical results for the masses and decay constants of the π , K mesons can reproduce the experimental values, the ground state masses of the scalar diquarks U^a , D^a , S^a are consistent with the existing theoretical calculations. We suggest a new Lagrangian which may explain the uncertainty of the masses of the scalar diquarks.

PACS : 14.40.-n, 11.10.Gh, 11.10.St, 12.40.qq

Key Words: Schwinger-Dyson equation, Bethe-Salpeter equation, diquark, confinement

1 Introduction

The discovery of the pentaquark state $\Theta^+(1540)$ has opened a new field of strong interaction and provides a new opportunity for a deeper understanding of the low energy QCD. Intense theoretical studies have been motivated to clarify the quantum numbers and to understand the under-structures of the pentaquark state $\Theta^+(1540)$ [1]. Although the existence of the $\Theta^+(1540)$ is uncertain and is a subject of controversy now, the $\Theta^+(1540)$ has already contributed to hadron spectroscopy. Unlike the chiral soliton model, the quark models take the constituent quarks or quark clusters as the elementary degrees of freedom, there exist a great number of possible quark configurations satisfy the Fermi statistics and the color singlet condition for the substructures of the pentaquark state $\Theta^+(1540)$ if we release stringent dynamical

¹Corresponding author; E-mail, wangzgyiti@yahoo.com.cn.

constraints. In fact, the multi-quark states are many-body problems, they are very difficult to solve. Whether or not the quarks can cluster together to form diquarks is of great importance theoretically, if we take the diquarks as the basic constituents (here the "basic constituents" does not mean they are asymptotic states, they just exist inside the baryons or multi-quark states with typical length), the problems will be greatly simplified, for example, the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet baryons can be taken as quark-diquark bound states in the Faddeev approximation [2], the nonet scalar mesons below 1GeV can be taken as 4-quark states $(qq)_3(\bar{q}\bar{q})_3$ with scalar diquarks or pseudoscalar diquarks as their basic constituents [3]; furthermore, we can obtain more insight into the relevant degrees of freedom and deepen our understanding about the underlying dynamics that determines the properties of the baryons and exotic multi-quark states. A typical quark model for the pentaquark states is the Jaffe-Wilczek's diquark-diquark-antiquark model [4]. In this model, the scalar diquarks $U^a = \epsilon^{abc}d_b^T(x)C\gamma_5 s_c(x)$, $D^a = \epsilon^{abc}u_b^T(x)C\gamma_5 s_c(x)$, $S^a = \epsilon^{abc}u_b^T(x)C\gamma_5 d_c(x)$ are taken as the basic constituents. They belong to the antitriplet $\bar{3}$ representation of both the color $SU(3)_c$ group and flavor $SU(3)_f$ group, in the color superconductivity theory, the attractive interactions in this channel lead to the formulation of nonzero condensates and breaking of both the color and flavor $SU(3)$ symmetries [5]. The scalar diquarks correspond to the 1S_0 states of the diquark systems, the one-gluon exchange force and the instanton induced force can lead to significant attractions between the quarks in the 0^+ channels [6]. The pseudoscalar diquarks do not have nonrelativistic limit, can be taken as the 3P_0 states. As the instanton induced force results in strong attractions in the scalar diquark channel and strong repulsions in the pseudoscalar diquark channel, if the effects of the instanton are manifested, we prefer the S type diquark to the P type diquark in constructing interpolating currents in the QCD sum rules [6].

In this article, we take the point of view that the scalar diquarks are quasi-bound states of quark-quark system and study the ground state mass spectrum within the framework of the coupled Schwinger-Dyson equation (SDE) and Bethe-Salpeter equation (BSE). The coupled rainbow SDE and ladder BSE have given a lot of successful descriptions of the long distance properties of the low energy QCD and the QCD vacuum (for reviews, one can see Refs.[7, 8, 9, 10]). The SDE can naturally embody the dynamical symmetry breaking and confinement which are two crucial features of QCD, although they correspond to two very different energy scales [11, 12]. On the other hand, the BSE is a conventional approach in dealing with the two-body relativistic bound state problems [13]. From the solutions of the BSE, we can obtain useful information about the under-structures of the mesons and diquarks, and obtain powerful tests for the quark theory. However, the obviously drawback may be the model dependent kernels for the gluon two-point Green's function and the truncations for the coupled divergent SDE and BSE series in one or the other ways [14]. Many analytical and numerical calculations indicate that the coupled rainbow SDE and ladder BSE with phenomenological potential models can give model independent results and satisfactory values [15, 16]. The usually used effective

potential models are confining Dirac δ function potential, Gaussian distribution potential and flat bottom potential (FBP) [17, 18, 19]. The FBP is a sum of Yukawa potentials, which not only satisfies chiral invariance and fully relativistic covariance, but also suppresses the singular point that the Yukawa potential has. It works well in understanding the dynamical chiral symmetry breaking, confinement and the QCD vacuum as well as the meson structures, such as electromagnetic form factors, radius, decay constants [14, 20, 21, 22]. In this article, we use the FBP to study the ground state mass spectrum of the scalar diquarks without fine tuning such as modifying the infrared behavior for the heavy quark systems.

The article is arranged as follows: we introduce the FBP in section II; in section III, IV and V, we solve the rainbow SDE and ladder BSE, explore the analyticity of the quark propagators, study the dynamical symmetry breaking and confinement, finally obtain the mass spectrum of the π , K mesons and the scalar U^a , D^a , S^a diquarks, and the decay constants of the π , K mesons; section VI is reserved for conclusion.

2 Flat Bottom Potential

The present techniques in QCD calculation can not give satisfactory large r behavior for the gluon two-point Green's function to implement the linear potential confinement mechanism, in practical calculation, the phenomenological effective potential models always do the work. The FBP is a sum of Yukawa potentials which is an analogy to the exchange of a series of particles and ghosts with different masses (Euclidean Form),

$$G(k^2) = \sum_{j=0}^n \frac{a_j}{k^2 + (N + j\rho)^2}, \quad (1)$$

where N stands for the minimum value of the masses, ρ is their mass difference, and a_j is their relative coupling constant. Due to the particular condition we take for the FBP, there is no divergence in solving the SDE. In its three dimensional form, the FBP takes the following form:

$$V(r) = - \sum_{j=0}^n a_j \frac{e^{-(N+j\rho)r}}{r}. \quad (2)$$

In order to suppress the singular point at $r = 0$, we take the following conditions:

$$\begin{aligned} V(0) &= \text{constant}, \\ \frac{dV(0)}{dr} &= \frac{d^2V(0)}{dr^2} = \dots = \frac{d^nV(0)}{dr^n} = 0. \end{aligned} \quad (3)$$

The a_j can be determined by solving the equations inferred from the flat bottom condition in Eq.(3). As in previous literature [14, 19, 20, 21, 22], n is set to be 9.

3 Schwinger-Dyson equation

The SDE can provide a natural framework for studying the nonperturbative properties of the quark and gluon Green's functions. By studying the evolution behavior and analytic structure of the dressed quark propagators, we can obtain valuable information about the dynamical chiral symmetry breaking and confinement. In the following, we write down the rainbow SDE for the quark propagator,

$$S^{-1}(p) = i\gamma \cdot p + \hat{m}_{u,d,s} + 4\pi \int \frac{d^4k}{(2\pi)^4} \gamma_\mu \frac{\lambda^a}{2} S(k) \gamma_\nu \frac{\lambda^a}{2} G_{\mu\nu}(k-p), \quad (4)$$

where

$$S^{-1}(p) = iA(p^2)\gamma \cdot p + B(p^2) \equiv A(p^2)[i\gamma \cdot p + m(p^2)], \quad (5)$$

$$G_{\mu\nu}(k) = (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})G(k^2), \quad (6)$$

and $\hat{m}_{u,d,s}$ stands for the current quark mass that breaks chiral symmetry explicitly. For a short discussion about the full SDE for the quark propagator, one can consult Ref.[20].

In this article, we assume that a Wick rotation to Euclidean variables is allowed, and perform a rotation analytically continuing p and k into the Euclidean region. Alternatively, one can derive the SDE from the Euclidean path-integral formulation of the theory, thus avoiding possible difficulties in performing the Wick rotation [23]. As far as only numerical results are concerned, the two procedures are equal. In fact, the analytical structures of quark propagators have interesting information about confinement, we will make detailed discussion about the propagators of the u , d and s quarks in section V.

4 Bethe-Salpeter equation

The BSE is a conventional approach in dealing with the two-body relativistic bound state problems [13]. The precise knowledge about the quark structures of the mesons and diquarks can result in better understanding of their properties. In the following, we write down the ladder BSE for the scalar diquark quasi-bound states with two quarks of different flavor [24, 25],

$$\Gamma_{\bar{3}}(q, P) = -4\pi \int \frac{d^4k}{(2\pi)^4} G_{\mu\nu}(q-k) \gamma_\mu \frac{\lambda^a}{2} S(k + \xi P) \Gamma_{\bar{3}}(k, P) S^T(-k + (1-\xi)P) (\gamma_\nu \frac{\lambda^a}{2})^T. \quad (7)$$

Here T denotes matrix transpose, the $S(k)$ is the quark propagator, $G_{\mu\nu}(k)$ is the gluon propagator, P_μ is the four-momentum of the center of mass of the scalar diquark, q_μ is the relative four-momentum between the two quarks, γ_μ is the bare

quark-gluon vertex, and $\Gamma_{\bar{3}}(q, P)$ is the Bethe-Salpeter amplitude of the quasi-bound state (or diquark). The ξ is the center of mass parameter which can be chosen to vary between 0 and 1, for the S^a diquark, $\xi = \frac{1}{2}$, for the U^a and D^a diquarks, as the current quark masses $m_s > m_u$ and $m_s > m_d$, ξ is about $\frac{1}{2}$. We can introduce an auxiliary amplitude $\Gamma_{\bar{3}}^C(q, P)$ to facilitate the calculation,

$$\Gamma_{\bar{3}}^C(q, P) \equiv \Gamma_{\bar{3}}(q, P) C, \quad (8)$$

here $C = \gamma_2 \gamma_4$ is the charge conjugation matrix. The auxiliary amplitude $\Gamma_{\bar{3}}^C(q, P)$ satisfies the following equation,

$$\Gamma_{\bar{3}}^C(q, P) = -\frac{8\pi}{3} \int \frac{d^4 k}{(2\pi)^4} G_{\mu\nu}(q - k) \gamma_\mu S(k + \xi P) \Gamma_{\bar{3}}^C(k, P) S(k - (1 - \xi)P) \gamma_\nu. \quad (9)$$

It is obviously the above equation is identical to the BSE for the pseudoscalar mesons but a reduction in the coupling strength, $\frac{4}{3} \rightarrow \frac{2}{3}$. We can introduce the Bethe-Salpeter wavefunction (BSW) χ_{qq} for the quasi-bound states,

$$\chi_{qq}(q, P) \equiv S(q + \xi P) \Gamma_{\bar{3}}^C(p, P) S(q - (1 - \xi)P), \quad (10)$$

to relate with our previously works on the pseudoscalar meson's BSE,

$$S^{-1}(q + \xi P) \chi_{qq}(q, P) S^{-1}(q - (1 - \xi)P) = -\frac{8\pi}{3} \int \frac{d^4 k}{(2\pi)^4} \gamma_\mu \chi_{qq}(k, P) \gamma_\nu G_{\mu\nu}(q - k), \quad (11)$$

$$S^{-1}(q + \xi P) \chi_{q\bar{q}}(q, P) S^{-1}(q - (1 - \xi)P) = -\frac{16\pi}{3} \int \frac{d^4 k}{(2\pi)^4} \gamma_\mu \chi_{q\bar{q}}(k, P) \gamma_\nu G_{\mu\nu}(q - k), \quad (12)$$

We can perform the Wick rotation analytically and continue q and k into the Euclidean region². The BSWs of the scalar diquarks χ_{qq} and pseudoscalar mesons $\chi_{q\bar{q}}$ have the same Dirac structures, can be signed by the notation $\chi(q, P)$. In the lowest order approximation, the BSW $\chi(q, P)$ can be written as

$$\chi(q, P) = \gamma_5 [iF_1^0(q, P) + \gamma \cdot P F_2^0(q, P) + \gamma \cdot qq \cdot P F_3^1(q, P) + i[\gamma \cdot q, \gamma \cdot P] F_4^0(q, P)] \quad (13)$$

In solving the BSEs, it is important to translate the wavefunctions F_i^n into the same dimension,

$$F_1^0 \rightarrow \Lambda^0 F_1^0, F_2^0 \rightarrow \Lambda^1 F_2^0, F_3^1 \rightarrow \Lambda^3 F_3^1, F_4^0 \rightarrow \Lambda^2 F_4^0, q \rightarrow q/\Lambda, P \rightarrow P/\Lambda,$$

here the Λ is some quantity of the dimension of mass. The ladder BSEs for the scalar diquarks and pseudoscalar mesons can be projected into the following four coupled integral equations,

$$\sum_j H(i, j) F_j^{0,1}(q, P) = \sum_j \int d^4 k K(i, j), \quad (14)$$

²To avoid possible difficulties in performing the Wick rotation, one can derive the BSE from the Euclidean path-integral formulation of the theory.

the expressions of the $H(i, j)$ and $K(i, j)$ are cumbersome and neglected here.

We can introduce a parameter $\lambda(P^2)$ and solve the above equations as an eigenvalue problem. If there really exist a quasi-bound state of two-quark, the masses of the diquarks can be determined by the condition $\lambda(P^2 = -M_{qq}^2) = 1$,

$$\sum_j H(i, j) F_j^{0,1}(q, P) = \lambda(P^2) \sum_j \int d^4k K(i, j). \quad (15)$$

Here we will take a short digression and give some explanations for the expressions of $H(i, j)$. The $H(i, j)$'s are functions of the quark's Schwinger-Dyson functions (SDF) $A(q^2 + \xi^2 P^2 + 2\xi q \cdot P)$, $B(q^2 + \xi^2 P^2 + 2\xi q \cdot P)$, $A(q^2 + (1 - \xi)^2 P^2 - 2(1 - \xi)q \cdot P)$ and $B(q^2 + (1 - \xi)^2 P^2 - 2(1 - \xi)q \cdot P)$. The relative four-momentum q is a quantity in the Euclidean spacetime while the center of mass four-momentum P must be continued to the Minkowski spacetime i.e. $P^2 = -m_{\pi, K, U^a, D^a, S^a}^2$, this results in that the $q \cdot P$ varies throughout a complex domain. It is inconvenient to solve the SDE with the resulting complex values of the quark momentum. We can expand the A and B in terms of Taylor series of $q \cdot P$, for example,

$$A(q^2 + \xi^2 P^2 + \xi q \cdot P) = A(q^2 + \xi^2 P^2) + 2\xi A(q^2 + \xi^2 P^2)' q \cdot P + \dots$$

The other problem is that we can not solve the SDE in the timelike region as the two-point gluon Green's function can not be exactly inferred from the $SU(3)$ color gauge theory even in the low energy spacelike region. In practical calculations, we can extrapolate the values of the A and B from the spacelike region smoothly to the timelike region with suitable polynomial functions. To avoid possible violation with confinement in sense of the appearance of pole masses $q^2 = -m^2(q^2)$ in the timelike region, we must be care in choosing the polynomial functions [18].

Finally we write down the normalization condition for the BSWs of the pseudoscalar mesons,

$$N_c \int \frac{d^4q}{(2\pi)^4} Tr \left\{ \bar{\chi} \frac{\partial S_+^{-1}}{\partial P_\mu} \chi(q, P) S_-^{-1} + \bar{\chi} S_+^{-1} \chi(q, P) \frac{\partial S_-^{-1}}{\partial P_\mu} \right\} = 2P_\mu, \quad (16)$$

here $\bar{\chi} = \gamma_4 \chi^\dagger \gamma_4$, $S_+ = S(q + \xi P)$ and $S_- = S(q - (1 - \xi)P)$. In this article, the parameters of FBP are fitted to give the correct masses and decay constants for the pseudoscalar mesons, π and K , the normalization condition is needed.

5 Coupled rainbow SDE and ladder BSE, and the mass spectrum

In this section, we study the coupled equations of the rainbow SDE and ladder BSE for the pseudoscalar mesons (π and K) and scalar diquarks (U^a , D^a and S^a) numerically, the final results for the SDFs and BSWs can be plotted as functions of the square momentum q^2 .

In order to demonstrate the confinement of quarks, we have to study the analyticity of the SDFs of the u , d and s quarks, and prove that there are no mass poles on the real timelike q^2 axial. In the following, we take the Fourier transform with respect to the Euclidean time T for the scalar part (S_s) of the quark propagator [7, 9, 26],

$$S_s^*(T) = \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4 T} \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)} \Big|_{\vec{q}=0}, \quad (17)$$

where the 3-vector part of q is set to zero. If $S(q)$ has a mass pole at $q^2 = -m^2(q^2)$ in the real timelike region, the Fourier transformed $S_s^*(T)$ would fall off as e^{-mT} for large T or $\log S_s^* = -mT$. In our numerical calculations, for small T , the values of S_s^* are positive and decrease rapidly to zero and beyond with the increase of T , which are compatible with the result (curve tendency with respect to T) from lattice simulations [27]; for large T , the values of S_s^* are negative, except occasionally a very small fraction of positive values. The negative values for S_s^* indicate an explicit violation of the axiom of reflection positivity [28], in other words, the quarks are not physical observable i.e. confinement. As colored quantity, the diquarks should also be confined and not appear as asymptotic states; the demonstration of their confinement is beyond the present work.

The u , d and s quarks have small current masses, the dressing or renormalization is very large and the curves of the SDFs are steep, which are corresponding to the dynamical chiral symmetry breaking phenomenon for the light quarks. At zero momentum, $m_u(0) = m_d(0) = 0.454 GeV$ and $m_s(0) = 0.684 GeV$, which are compatible with the constituent quark masses in the literature. From the solutions of BSEs for the π , K mesons and U^a , D^a , S^a diquarks as eigenvalue problems, we can obtain the masses for those pseudoscalar mesons and scalar diquarks,

$$\begin{aligned} M_\pi &= 135 MeV, \quad M_K = 498 MeV, \\ M_{S^a} &= 0.76 GeV, \quad M_{U^a} = 0.98 GeV, \quad M_{D^a} = 0.98 MeV. \end{aligned} \quad (18)$$

It is obviously

$$M_{S^a} < m_u(0) + m_d(0), \quad M_{U^a} < m_d(0) + m_s(0), \quad M_{D^a} < m_u(0) + m_s(0), \quad (19)$$

$$M_{U^a} - M_{S^a} \approx m_s(0) - m_u(0) \approx 0.22 GeV. \quad (20)$$

The attractive interaction between the quarks in the color and flavor $SU(3) \bar{3}$ channel can lead to the quasi-bound states in the infrared region. The appearance of the diquarks is closely related to the dynamical symmetry breaking phenomenon, the mass splitting among the U^a , D^a and S^a diquarks originate from the mass splitting among u , d and s quarks.

The existing theoretical calculations for the masses of the scalar diquarks vary in a large range, $M_{qq} = (0.4 - 0.7) GeV$ with QCD sum rules [29]; $M_{qq} \sim 0.5 GeV$ with random instanton liquid model [30]; $M_{qq} = (0.42 \pm 0.03) GeV$ with random instanton

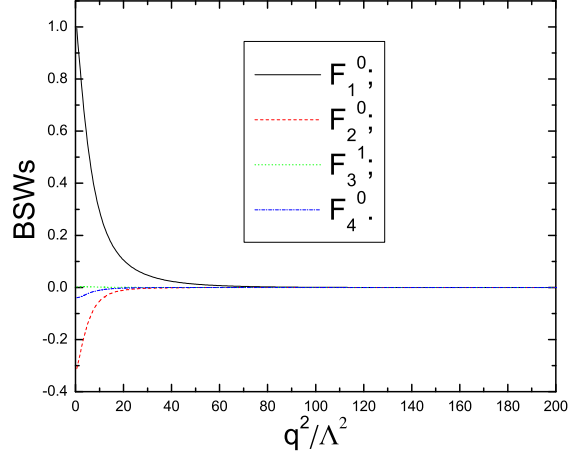


Figure 1: Un-normalized BSWs for the π meson.

liquid model [31], $M_{qq} = 0.234\text{GeV}$ with Nambu-Jona-Lasinio Model [32]; $M_{S^a} = 0.74\text{GeV}$, $M_{U^a} = M_{D^a} = 0.88\text{GeV}$ with BSE [25]; $M_{S^a} = 0.82\text{GeV}$, $M_{U^a} = M_{D^a} = 1.10\text{GeV}$ with BSE [33]; $M_{qq} = 0.692\text{GeV}$ with global color model [34]; $0.14 < M_{qq} < 0.74\text{GeV}$ with assumption of mass functions [24]; $M_{qq} \approx 0.7\text{GeV}$ with lattice QCD [35]. There are large uncertainties for the masses of the quasi-bound states, U^a , D^a , and S^a . As colored quantities, the diquarks may have gauge interactions with the gluon field as fundamental scalar field with the following Lagrangian,

$$L = -\frac{1}{2}D_\mu S^a D_\mu S^a - \frac{1}{2}M_{S^a}^2 S^a S^a, \quad (21)$$

$$D_\mu S^a = \partial_\mu S^a + ig f^{abc} A_\mu^b S^c. \quad (22)$$

Here we use the notation S^a to represent the scalar diquark field and A_μ^b the gluon field. The direct color interactions between the scalar diquarks and the gluons may modify the mass M_{qq} significantly.

From the plotted BSWs (see Fig.1 for the π meson and Fig.2 for the S^a diquark as examples), we can see that the BSWs of the pseudoscalar mesons and scalar diquarks have the same type (Gaussian type) momentum dependence while the quantitative values are different from each other. Just like the $\bar{q}q$, $\bar{q}Q$ and $\bar{Q}Q$ pseudoscalar mesons [14, 20], the gaussian type BSWs of the scalar diquarks center around zero momentum and extend to the energy scale about $q^2 = 1\text{GeV}^2$ which happen to be the energy scale for the chiral symmetry breaking, the strong interactions in the infrared region result in quasi-bound states, U^a , D^a and S^a . The BSWs of the π and K mesons can give satisfactory values for the decay constants which are defined

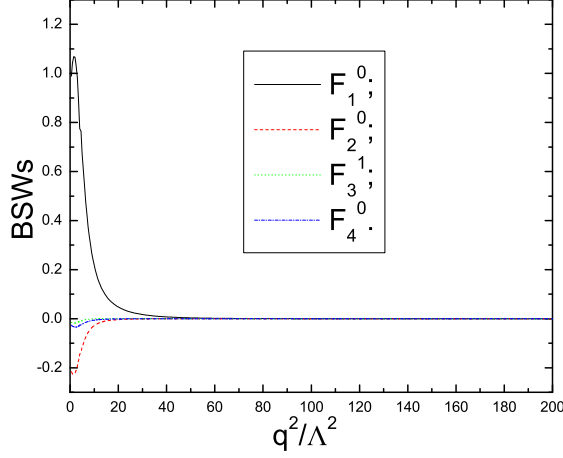


Figure 2: Un-normalized BSWs for the S^a diquark .

by

$$\begin{aligned}
 i f_\pi P_\mu &= \langle 0 | \bar{q} \gamma_\mu \gamma_5 q | \pi(P) \rangle, \\
 &= N_c \int Tr [\gamma_\mu \gamma_5 \chi(k, P)] \frac{d^4 k}{(2\pi)^4},
 \end{aligned} \tag{23}$$

here we use π to represent the pseudoscalar mesons,

$$f_\pi = 127 MeV; \quad f_K = 161 MeV. \tag{24}$$

There are negative voice for the existence of the scalar diquark states in the infrared region [36]. The coupled rainbow SDE and ladder BSE are particularly suitable for studying the flavor octet pseudoscalar mesons and vector mesons, the next-to-leading-order (NLO) contributions from the quark-gluon vertex have a significant amount of cancellation between repulsive and attractive corrections. However, for the diquarks and scalar mesons, the large repulsive corrections from the NLO contributions can significantly change the scalar meson masses and corresponding BSWs; the diquark quasi-bound states found in the ladder BSE will disappear from the spectrum. In this article, we do not take the point of view that the scalar diquarks exist in the strong interaction spectrum as asymptotic states, and our conclusion do not conflict with Ref.[36]; the confinement precludes the observation of the free colored diquarks, the quark-quark can correlate with each other in the color and flavor $\bar{3}$ channels inside the baryons and multi-quark states with typical length $l = \frac{1}{M_{qq}}$. In fact, the NLO contributions from the quark-gluon vertex are extremely difficult to take into account if we go beyond the infrared dominated δ function approximation for the gluon kernel, which is obviously violate the lorentz invariance; the lorentz invariant and model-independent treatments still lack in the literatures.

In calculation, the values of current quark masses are taken as $\hat{m}_u = \hat{m}_d = 6MeV$ and $\hat{m}_s = 150MeV$; the input parameters for the FBP are $N = 1.0\Lambda$, $V(0) = -17.0\Lambda$, $\rho = 6.0\Lambda$ and $\Lambda = 200MeV$, which are fitted to give the correct masses of the π and K mesons. In this article, we deal with only the light flavor quarks, the FBP can give satisfactory results without fine tuning, such as modifying the infrared behavior for the heavy quarks c and b .

6 Conclusion

In this article, we study the under-structures of the pseudoscalar mesons π , K and scalar diquarks U^a , D^a , S^a in the framework of the coupled rainbow SDE and ladder BSE with the confining effective potential (FBP). After we solve the coupled rainbow SDE and ladder BSE numerically, we obtain the SDFs and BSWs of the pseudoscalar mesons π , K and scalar diquarks U^a , D^a , S^a , and the corresponding ground state mass spectrum. The u , d and s quarks have small current masses, the dressing or renormalization for the SDFs is very large and the curves are steep which indicate the dynamical chiral symmetry breaking phenomenon for the light quarks explicitly. The mass poles in the timelike region are absent which implement the confinement naturally. The BSWs of the pseudoscalar mesons and scalar diquarks have the same type (Gaussian type) momentum dependence while the quantitative values are different from each other. The gaussian type BSWs center around zero momentum and extend to the energy scale about $q^2 = 1GeV^2$ which happen to be the energy scale for the chiral symmetry breaking, the strong interactions in the infrared region result in bound (or quasi-bound) states. Our numerical results for the masses and decay constants of the π , K mesons can reproduce the experimental values, the ground state mass spectrum of scalar diquarks are consistent with the existing theoretical calculations. The mass splitting among the U^a , D^a and S^a diquarks originate from the mass splitting among u , d and s quarks. $M_{S^a} < m_u(0) + m_d(0)$, $M_{U^a} < m_d(0) + m_s(0)$, $M_{D^a} < m_u(0) + m_s(0)$, the attractive interaction between the color and flavor $SU(3)$ $\bar{3}$ quarks can lead to the quasi-bound states in the infrared region. Once the satisfactory SDFs and BSWs of the scalar diquarks are known, we can use them to study a lot of important quantities involving the multi-quark states.

Acknowledgment

This work is supported by National Natural Science Foundation, Grant Number 10405009, and Key Program Foundation of NCEPU. The authors are indebted to Dr. J.He (IHEP), Dr. X.B.Huang (PKU) and Dr. L.Li (GSCAS) for numerous help, without them, the work would not be finished. The authors would also like to thank Prof. C. D. Roberts for providing us some important literatures.

References

- [1] M. Oka, Prog. Theor. Phys. **112** (2004) 1; S. L. Zhu, Int. J. Mod. Phys. **A19** (2004) 3439; S. L. Zhu, hep-ph/0410002; F. E. Close, hep-ph/0311087; B. K. Jennings and K. Maltman, Phys. Rev. **D69** (2004) 094020; and references therein.
- [2] M. Oettel, nucl-th/0012067; A. Hoell, C. D. Roberts, S. V. Wrightand, nucl-th/0601071; and references therein.
- [3] T. V. Brito, F. S. Navarra, M. Nielsen, M. E. Bracco, Phys. Lett. **B608** (2005) 69; Z. G. Wang, W. M. Yang, Eur. Phys. J. **C42** (2005) 89; Z. G. Wang, W. M. Yang, S. L. Wan, J. Phys. **G31** (2005) 971.
- [4] R. Jaffe and F. Wilczek, Phys. Rev. Lett. **91** (2003) 232003.
- [5] K. Rajagopal, F. Wilczek, hep-ph/0011333; D. K. Hong, hep-ph/0101025; T. Schafer, hep-ph/0304281; and references therein.
- [6] A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. **D12** (1975) 147; T. DeGrand, R. L. Jaffe, K. Johnson and J. E. Kiskis, Phys. Rev. **D12** (1975) 2060; G. 't Hooft, Phys. Rev. **D14** (1976) 3432 [Erratum-ibid. Phys. Rev. **D18** (1978) 2199]; E. V. Shuryak, Nucl. Phys. **B203** (1982) 93; T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70** (1998) 323; E. Shuryak and I. Zahed, Phys. Lett. **B 589** (2004) 21.
- [7] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477.
- [8] P. C. Tandy, Prog. Part. Nucl. Phys. **39** (1997) 117.
- [9] C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45** (2000) S1.
- [10] C. D. Roberts, nucl-th/0304050; P. Maris and C. D. Roberts, Int. J. Mod. Phys. **E12** (2003) 297.
- [11] V. A. Miransky, Dynamical Symmetry Breaking in Quantum Field Theories, Word Scientific, 1993.
- [12] R. Alkofer and L. V. Smekal, Phys. Rept. **353** (2001) 281; C. S. Fischer and R. Alkofer, hep-ph/0301094.
- [13] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84** (1951) 1232; N. Nakanishi, Suppl. Prog. Theor. Phys. **43** (1969) 1.
- [14] Z. G. Wang, W. M. Yang and S. L. Wan, Phys. Lett. **B584** (2004) 71; Z. G. Wang , W. M. Yang and S. L. Wan, Nucl. Phys. **A744** (2004) 156.

- [15] Y. B. Dai, C. S. Huang, D. S. Liu, Phys. Rev. **D43** (1991) 1717; Y. B. Dai, Y. B. Ding, C. S. Huang, C. L. Wang, Y. L. Zhu, Phys. Rev. **D47** (1993) 1256.
- [16] For example, P. Maris and C. D. Roberts, Phys. Rev. **C56** (1997) 3369; P. Maris, C. D. Roberts and P. C. Tandy, Phys. Lett. **B420** (1998) 267; P. Maris and P. C. Tandy, Phys. Rev. **C60** (1999) 055214; P. Maris, Nucl. Phys. **A663** (2000) 621; M. A. Ivanov, Yu. L. Kalinovsky and C. D. Roberts, Phys. Rev. **D60** (1999) 034018; M. A. Ivanov, Yu. L. Kalinovsky, P. Maris and C. D. Roberts, Phys. Lett. **B416** (1998) 29; M. A. Ivanov, Yu. L. Kalinovsky, P. Maris and C. D. Roberts, Phys. Rev. **C57** (1998) 1991.
- [17] H. J. Munczek and A. M. Nemirovsky, Phys. Rev. **D28** (1983) 181.
- [18] H. J. Munczek and P. Jain, Phys. Rev. **D46** (1991) 438; P. Jain and H. J. Munczek, **D44** (1991) 1873; P. Jain and H. J. Munczek, **D48** (1993) 5403.
- [19] K. L. Wang and S. L. Wan, Phys. Rev. **D47** (1993) 2098.
- [20] Z. G. Wang, W. M. Yang, S. L. Wan, Phys. Lett. **B615** (2005) 79.
- [21] Z. G. Wang, S. L. Wan and K. L. Wang, Phys. Lett. **B498** (2001) 195; Z. G. Wang, J. Phys. **G28** (2002) 3007; Z. G. Wang, S. L. Wan and K. L. Wang, Commun. Theor. Phys. **35** (2001) 697; S. L. Wan and K. L. Wang, J. Phys. **G22** (1996) 1287.
- [22] Z. G. Wang and S. L. Wan, Phys. Lett. **B536** (2002) 241.
- [23] S. T. Stainsby and R. T. Cahill, Phys. Lett. **146A** (1990) 467.
- [24] R. T. Cahill, C. D. Roberts, J. Praschifka, Phys. Rev. **D36** (1987) 2804.
- [25] C. J. Burden, Q. Lu, C. D. Roberts, P. C. Tandy, M. J. Thomson, Phys. Rev. **C55** (1997) 2649.
- [26] P. Maris, Phys. Rev. **D52** (1995) 6087.
- [27] M. S. Bhagwat, M. A. Pichowsky, C. D. Roberts and P. C. Tandy, Phys. Rev. **C68** (2003) 015203.
- [28] J. Glimm and A. Jaffee, Quantum Physics. A Functional Point of View (Springer-Verlag, New York, 1981).
- [29] H.G. Dosch, M. Jamin and B. Stech, Z. Phys. **C42**(1989) 167.
- [30] M. Cristoforetti, P. Faccioli, G. Ripka and M. Traini, hep-ph/0410304.
- [31] T. Schaefer, E. V. Shuryak, J. Verbaarschot, Nucl. Phys. **B412** (1994) 143.
- [32] U. Vogl, Z. Phys. **A337** (1990) 191.

- [33] P. Maris, Few Body Syst. **32** (2002) 41.
- [34] R. T. Cahill, S. M. Gunner, Phys. Lett. **B359** (1995) 281.
- [35] M. Hess, F. Karsch, E. Laermann, I. Wetzorke, Phys. Rev. **D58** (1998) 111502.
- [36] A. Bender, W. Detmold, C. D. Roberts, A. W. Thomas Phys. Rev. **C65** (2002) 065203; A. Bender, C. D. Roberts, L. Von Smekal, Phys. Lett. **B380** (1996) 7; G. Hellstern, R. Alkofer, H. Reinhardt, Nucl. Phys. **A625** (1997) 697; M. S. Bhagwat, A. Holl, A. Krassnigg, C. D. Roberts, P.C. Tandy, Phys. Rev. **C70** (2004) 035205.